Seat No. : _______________

N17-124

November-2014

B.Sc., Sem.-V

305 : Mathematics (Elective)

(Discrete Mathematics)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : (1) **All** the questions are compulsory.

(2) Figures to the right indicate full marks for the question.

1. (a) Define partial order relation and poset with suitable example. Give an example of a poset, which is not a lattice with proper justification. Give an example of an infinite bounded lattice with proper justification. **9**

OR

Show that in a lattice $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

(b) Show that in a lattice if $a \leq b \leq c$, then $a \oplus b = b * c$ and $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$. **9**

OR

Show that in a lattice $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

2. (a) Define distributive lattice with a suitable example. Show that $(R, \leq)$ is a distributive lattice. **9**

OR

Define lattice homomorphism. Prove that the lattices $(P(X), \subset)$, with $X = \{a, b\}$ and (S_6, D) are homomorphic to each other.

(b) Prove that in a Boolean algebra $a = 0 \Leftrightarrow ab' + a'b = b$. **9**

OR

State and prove one of the De'Morgan's laws of a Boolean algebra.

3. (a) If m_i and m_j are two distinct minterms then prove that $m_i * m_j = 0$. How many minterms are there for n variables ? **9**

OR

Obtain the SOP and POS canonical forms of the Boolean expression $\alpha(x_1, x_2, x_3) = (x_1 \oplus x_2 \oplus x_3) \oplus (x_2' * x_3)$.

(b) State Stone's representation theorem for Boolean algebra and explain by a suitable example. Does there exists a Boolean algebra with exactly 2014 elements ? Explain. **9**

OR

Define a symmetric Boolean expression. Does the Boolean expression $f(a, b, c) = (abc') + (bca') + (cba')$ a symmetric Boolean expression ? Explain.

4. Answer in short : **16**

(a) Show that in a lattice $a * (a \oplus b) = a$.

(b) Give an example of a bounded finite lattice which is complemented but not distributive.

(c) Give an example of a bounded infinite lattice which is distributive but not complemented.

(d) Give an example of a bounded finite lattice which is non-distributive and non-complemented.

(e) Give an example of a lattice which is distributive and modular.

(f) Give an example of a lattice which is not Boolean algebra.

(g) Draw the Hasse Diagram of (S_{35}, D).

(h) Draw the Hasse Diagram of (S_{40}, D).

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N17-124

November-2014

B.Sc., Sem.-V

305 : Mathematics (Elective)

(Number Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : All questions are compulsory.

1. (a) State and prove the necessary sufficient condition for linear Diophantine equation has solution. Also solve in N : $25x + 35y = 145$. **9**

OR

Find integers x, y, z such that $\gcd(198, 288, 512) = 198x + 288y + 512z$.

(b) State and prove the Division Algorithm theorem for integers. **9**

OR

(i) If 792 divides the integer $13xy45z$ then find the digits x, y and z.

(ii) Prove that if a & b are both odd integers then $16 \mid a^4 + b^4 - 2$.

2. (a) State and prove the fundamental theorem of arithmetic. **9**

OR

Solve : $17x \equiv 9 (\text{mod} 276)$

(b) Prove that there are infinitely many primes of the form $4k + 3$. **9**

OR

(i) Prove : $27 \mid 2^{5n+1} + 5^{n+2}$, $\forall\ n \geq 1$

(ii) Verify : $89 \mid 2^{44} - 1$

3. (a) State and prove the Wilson's theorem and show that $18! \equiv -1 (\text{mod } 437)$ **9**

OR

Find r : $53^{103} + 103^{53} \equiv r (\text{mod} 39)$, where $0 \leq r < 39$.

(b) Prove : The quadratic congruence $x^2 + 1 \equiv 0 (\text{mod } p)$, where p is an odd prime has solution if and only if $p \equiv 1 (\text{mod } 4)$ **9**

OR

(i) Show that $18! \equiv -1 (\text{mod } 437)$

(ii) Find the remainder when $2(26!)$ is divided by 29.

4. Give the answer of following questions : **16**

(1) State the Fermat's little theorem.

(2) State the "Well Ordering Principle".

(3) Find the unit digit of 5^{81}.

(4) Find $\phi(525)$ & $\phi(2011)$.

(5) Find a prime of the form $n^2 - 4$.

(6) 50! end with how many zero ?

(7) Define g.c.d. of two integers.

(8) Is 1111998899 divisible by 7, 11 and 13 ? Justify.